Detection of switching events in the circuits using wavelet decomposition

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Abstract – The paper presents the application of discrete wavelet transform to the detection of switching events in electrical circuits. The wavelet decomposition is applied to the registered transient signal of the system. On the basis of the results of such decomposition the accurate determination of the time point of the switching event is determined. The paper presents the application of this approach to the detection of fault occurrence in the power transmission system. The numerical results concerning 400kV power transmission line are presented and discussed in the paper.

1 INTRODUCTION

The recognition and localization of the switching event in the system on the basis of the measured waveform is a very difficult but very useful task. Recognition of this point is important for example in the power system relaying systems, at the determination of the placement as well as the time point of occurrence of the fault on the basis of the registered voltage and current at one terminal [1,7]. The difficulty of this task is due to the fact that in spite of the fault, the transient curves can be still smooth and the breaking point hardly visible or recognized only as the abrupt change of the slope (the first derivative) of the waveform and not as the abrupt change of the waveform itself. The problem is drastically difficult for noisy waveform, since in this case simple differentiation is not a good tool for the recognition of the breaking point. Moreover the switching event in the loaded system may be done through high resistance and the results of such switching are practically invisible on the transient waveform.

The paper presents the discrete wavelet transform approach to the detection of the time point of the switching event that has happened in the electrical circuit. It is based on the observation of the distribution of the detailed wavelet coefficients on some decomposition levels. The moment of switching may be invisible on the original waveform, but is reflected on these levels as the abrupt change of their coefficient values.

The results of numerical experiments concerning the application of wavelets to the detection of the switching event in the electrical power systems will be presented and discussed in the paper. The switching events are regarded here as the results of the short-circuit fault occurring somewhere in the line through the unknown, possibly very high resistance.

2 WAVELET DECOMPOSITION

The discrete wavelet transform belongs to the multiresolution analysis [2,3,4]. It is a linear transformation with a special property of time and frequency localization at the same time. It decomposes the given signal onto a set of basis functions of different frequencies, shifted each other, called wavelets. Unlike DFT, the discrete wavelet transform is not a single object. In reality it hides a whole family of transformation. The individual members of the family are determined by the choice of so called mother wavelet function. The goal of wavelet transform is to decompose arbitrary signal f(t) into finite summation of wavelets at different scales (levels) according to the expansion

$$f(t) = \sum_{j} \sum_{k} c_{jk} \psi(2^{j}t - k)$$
(1)

where c_{jk} is a new set of coefficients and $\psi(2^j t - k)$ is the wavelet of jth level (scale) shifted by k samples. The set of wavelets of different scales and shifts can be generated from the single prototype wavelet, called mother wavelet, by dilations and shifts. What makes wavelet bases interesting is their self-similarity: every function in wavelet basis is a dilated and shifted version of one (or possibly few) mother functions. Once we know about the mother function we know everything about the basis. In practice the most often used are the orthogonal or bio-orthogonal wavelets, for which the set of wavelets forms an orthogonal or biorthogonal base.

The notion of wavelet is strictly associated with the so-called scaling function ϕ generated recursively according to the recurrence relation $\phi_{2j}(t) = 2^j \phi(2^j t)$. These functions for different levels of *j* form also orthogonal basis. In general they represent the low pass filtering of the signal. On the other hand the dilated wavelet functions can be associated with the band pass filtering. Let us denote the vector of the discrete form representation of the original signal f(t) by **f**. Let A_{2j} **f**

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denotes the operator that computes the approximation of **f** at resolution 2^{j} i.e.,

$$A_{2^{j}}\mathbf{f} = \left\langle \mathbf{f}, \phi_{2^{j}} \left(u - 2^{j} n \right) \right\rangle \tag{2}$$

where the symbol $\langle \ \rangle$ means the inner product,

 $\langle f, \phi \rangle = \int_{-\infty}^{\infty} f(u)\phi(u)du$ and ϕ - the impulse response of the special low pass filter. Let $D_{\gamma j}$ **f** denotes the

detailed signal, i.e., $D_{2^j}\mathbf{f} = A_{2^{j+1}}\mathbf{f} - A_{2^j}\mathbf{f}$ at the resolution 2^j , where this detailed signal can be determined also as the inner product

$$D_{2^{j}}\mathbf{f} = \left\langle \mathbf{f}, \boldsymbol{\psi}_{2^{j}} \left(u - 2^{j} n \right) \right\rangle \tag{3}$$

As was shown by Mallat [2] both operations can be interpreted as the convolution of the signal of previous resolution and the impulse response of the quadrature mirror filters: the high pass (\tilde{g}) and low pass (\tilde{h})

$$\begin{split} A_{2j}\mathbf{f} &= \sum_{k=-\infty}^{\infty} \widetilde{h}(2n-k)A_{2j+1}\mathbf{f}(2n) \\ D_{2j}\mathbf{f} &= \sum_{k=-\infty}^{\infty} \widetilde{g}(2n-k)A_{2j+1}\mathbf{f}(2n) \end{split}$$

These two operations performed for different values of j, j = -1, -2, ..., -J, deliver the coefficients of decomposition at different levels (scales) and form the analysis of the signal, known as Mallat pyramid algorithm. One stage of such analysis is presented in Fig. 1. The symbol $\downarrow 2$ means "keep one sample out of two". The notations \tilde{G} and \tilde{H} represent here the high pass (\tilde{G}) and low pass (\tilde{H}) analysis filters.



Figure 1: The Mallat pyramid algorithm, presenting one step of the decomposition of the signal.

As a result of such transformation we get the set of coefficients representing the detailed signals at different levels j, j = -J, -J+1, ..., -1 and the residue signal $A_{2^{-J}}\mathbf{f}$ at the level J. In this way N discrete values forming vector \mathbf{f} are represented by N wavelet coefficients. The coefficients of $D_{2^{j}}\mathbf{f}$ can be interpreted as the high frequency details that distinguish the approximation of \mathbf{f} at two subsequent levels of resolution. On the other hand the signal $A_{2^{-J}}\mathbf{f}$ represents the coarser approximation of the given original vector \mathbf{f} .



Figure 2: The Mallat pyramid algorithm of the reconstruction of the signal.

The wavelet decomposition is reversible. The inverse operation called reconstruction or synthesis, recovers the original signal representation at different resolutions using known wavelet coefficients. One step of Mallat pyramid algorithm is shown in Fig. 2. The symbol $\uparrow 2$ means "put one zero between each sample" and *2 – the multiplication by two. The notations H and G represent here the high pass (G) and low pass (H) synthesis filters. All filters: G, H, \tilde{G} and \tilde{H} form the quadrature mirror filters, whose impulse responses satisfy the following relations

$$g(n) = (-1)^{n} h(1-n)$$
$$\tilde{h}(n) = h(-n)$$
$$\tilde{g}(n) = g(-n)$$

All of them are defined on the basis of the assumed impulse response h(n) characteristic for particular type of wavelet. Choosing different forms of these filters we get different wavelet representations. To the most known discrete wavelets belong Haar, Daubechies, Lemarie, coiflets, symlets, etc [2,3,4].

3 PRINCIPLE OF SWITCHING EVENT DETECTION

Wavelet transform decomposes the measured waveform into different level basis functions. The detailed coefficients of each level represent the details of particular representation, from very fast (the decomposition levels closest to the original signal) to slow (the lowest levels of decomposition). So the levels closest to the original waveform represent the highest frequencies, contained in the signal, for example high frequency noise. This is exploited in the de-noising of the waveform, by reconstructing it with the highest-level coefficients equal to zero [3].

The next levels represent some inherent features characterizing the intermediate frequencies of the signal, among them the frequencies followed by the switching events. Observe that switching events change the transient signals of the network, but these changes are relatively slow, following from the existence of inductances and capacitances in the circuit. In many cases the change of character of the transient signal might be hardly visible, but usually is well reflected on the values of the succeeding coefficients of some intermediate levels of decomposition. At the moment of switching event the wavelet detailed coefficient of this levels change abruptly their values and in this way indicate the switching moment in the system under observation. To make the procedure independent on the absolute values of these coefficients, the difference of two neighboring samples (its absolute value) is taken into account. The position of the first value above the assumed threshold points to the moment of switching events.

4 THE RESULTS OF THE NUMERICAL EXPERIMENTS

The numerical experiments checking the ability of the proposed procedure of the switching event detection have been performed on the transient currents in the 400kV power transmission system, simulated using ATP-EMTP program [6]. We consider the power transmission line connecting two systems A and B as shown in Fig. 3 with the fault occurring somewhere in the transmission line.



Figure 3: The power system arrangement in the experiments.

The fault of the line is understood as the shorting of it by the resistance of the unknown value, occurring at the unknown distance [7]. Different types of faults have been simulated, including: phase-to-ground (Rg, S-g, T-g), phase-to-phase (R-S, S-T,R-T), twophases-to-ground (R-S-g, S-T-g, R-T-g) and threephase (R-S-T) fault. All numerical experiments of wavelet decomposition have been done using wavelet toolbox of Matlab [5].

Fig. 4 illustrates the wavelet decomposition method to the detection of the switching event in the power system on the basis of the analysis of the measured current waveform. The Haar wavelets have been applied in the experiments. The upper curve (Fig. 4a) represents the measured waveform, corresponding to some transient phenomena in the power system following the short-circuit fault. The next (Fig. 4b) illustrates the distribution of the 6th level wavelet

coefficients. There is a visible change of their magnitude corresponding to the moment of fault occurrence.



Figure 4: The illustration of different phases of detection of the switching event.

Fig. 4c depicts the absolute values of the difference of the consecutive coefficients. There is a visible point of rapid change of their values, which is associated with the switching event. The position of the first point above the assumed threshold indicates the switching event (Fig. 4d).



basis of the noisy waveform.

The next experiment shows how the method is insensitive to the noise. For this purpose we have disturbed the originally registered waveform by the 20dB white noise. Fig. 5a presents the actual disturbed measured waveform from previous experiment and Fig. 5b - the wavelet coefficient distribution on the sixth level. The described above procedure has recognized accurately the time of switching event without any problem (Fig. 5c,d) irrespective of the noise.

The method is also well suited to the multiple switching event detection. This will be illustrated by the next experiment. Fig. 6 illustrates the performance of the multiple switching event recognition occurring in the power system. Two switching events occurring at different time moments have been simulated in the 400kV power system. The first one corresponds to the one-phase short-circuit of the highly loaded system by using the high value resistance of 300 ohm. The next switching corresponds to the change of this shorting resistance to the value of 100 ohm. Both changes are practically invisible on the registered transient waveform (Fig. 6a). Although the changes of the registered waveform are invisible, the wavelet decomposition on 6th level (Fig. 6b) shows significant difference of the succeeding coefficient values before and after fault occurrence. This is especially well visible in the distribution of difference values of two succeeding coefficients (Fig. 6c). The further processing of these wavelet coefficients difference in accordance with the described procedure has allowed detecting both switching moments correctly (Fig. 6d).



Figure 6: The recognition of the multiple switching events on the basis of the measured waveform of the one-terminal current.

Observe that the proposed procedure allows also controlling the sensitivity of this detection process by applying different thresholds. Increasing the threshold value we can neglect small changes occurring in the system. In this way only important changes due to significant switching events can be detected.

5 CONCLUSIONS

The paper has proposed the wavelet-based approach to the precise detection of the switching event in the circuit on the basis of the registered transient signal. It has been proved, that the wavelet decomposition forms a very good basis for the localization of the breaking point of the time waveform due to some abrupt changes in the electrical circuit, followed for example by the fault occurring in it.

The features represented by the wavelet coefficients of some selected levels allow to indicate this point very precisely and to achieve much better accuracy than that obtained from the direct time analysis of the original waveform. The proposed method can find unique application in the power systems for fault recognition and localization, especially in the cases where the fault occurring in the system is hardly observable on the original or noisy transient curves registered at the origin of the transmission line.

Acknowledgments

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